OCR RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715820613

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 An empty railway truck of mass m_0 is moving along a straight horizontal track at speed v_0. The point P is at the front of the truck. The horizontal forces on the truck are negligible. As P passes a fixed point O, sand starts to fall vertically into the truck at a constant mass rate k. At time t after P passes O the speed of the truck is v and OP $= x$.

(i) Find an expression for v in terms of m_0, v_0, k and t, and show that $x = \dfrac{m_0 v_0}{k}\ln\left(1 + \dfrac{kt}{m_0}\right)$. **[9]**

(ii) Find the speed of the truck and the distance OP when the mass of sand in the truck is $2m_0$. **[2]**

2 A uniform rod AB of length 0.5 m and mass 0.5 kg is freely hinged at A so that it can rotate in a vertical plane. Attached at B are two identical light elastic strings BC and BD each of natural length 0.5 m and stiffness 2 N m^{-1}. The ends C and D are fixed at the same horizontal level as A and with AC = CD = 0.5 m. The system is shown in Fig. 2.1 with the angle BAC $= \theta$. You may assume that $\frac{1}{3}\pi \leqslant \theta \leqslant \frac{5}{3}\pi$ so that both strings are taut.

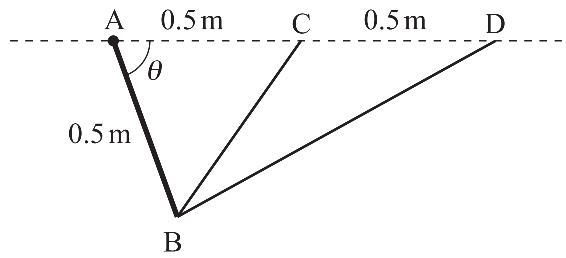

Fig. 2.1

(i) Show that the length of BC in metres is $\sin\frac{1}{2}\theta$. **[1]**

(ii) Find the potential energy, V J, of the system relative to AD in terms of θ. Hence show that

$$\frac{dV}{d\theta} = 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25 - \cos\theta}} - 0.5\cos\frac{1}{2}\theta.$$

[8]

(iii) Fig. 2.2 shows a graph of the function $f(\theta) = 1.5\sin\theta - 1.225\cos\theta - \dfrac{0.5\sin\theta}{\sqrt{1.25 - \cos\theta}} - 0.5\cos\frac{1}{2}\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

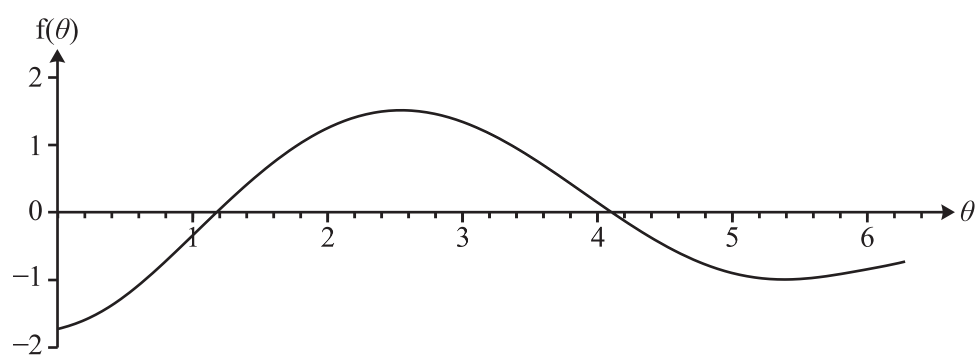

Fig. 2.2

Use the graph both to estimate, correct to 1 decimal place, the values of θ for which the system is in equilibrium and also to determine their stability. **[4]**

Section B (48 marks)

3 A model car of mass 2 kg moves from rest along a horizontal straight path. After time t s, the velocity of the car is $v\,\mathrm{m\,s^{-1}}$. The power, P W, developed by the engine is initially modelled by $P = 2v^3 + 4v$. The car is subject to a resistance force of magnitude $6v$ N.

(i) Show that $\frac{dv}{dt} = (1 - v)(2 - v)$ and hence show that $t = \ln\frac{2 - v}{2(1 - v)}$. **[10]**

(ii) Hence express v in terms of t. **[2]**

Once the power reaches 4.224 W it remains at this constant value with the resistance force still acting.

(iii) Verify that the power of 4.224 W is reached when $v = 0.8$ and calculate the value of t at this instant. **[2]**

(iv) Find v in terms of t for the motion at constant power. Deduce the limiting value of v as $t \to \infty$. **[10]**

4 A uniform lamina of mass m is in the shape of a sector of a circle of radius a and angle $\frac{1}{3}\pi$. It can rotate freely in a vertical plane about a horizontal axis perpendicular to the lamina through its vertex O.

(i) Show by integration that the moment of inertia of the lamina about the axis is $\frac{1}{2}ma^2$. **[6]**

(ii) State the distance of the centre of mass of the lamina from the axis. **[1]**

The lamina is released from rest when one of the straight edges is horizontal as shown in Fig. 4.1. After time t, the line of symmetry of the lamina makes an angle θ with the downward vertical.

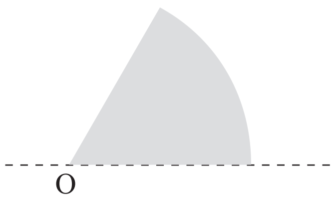

Fig. 4.1

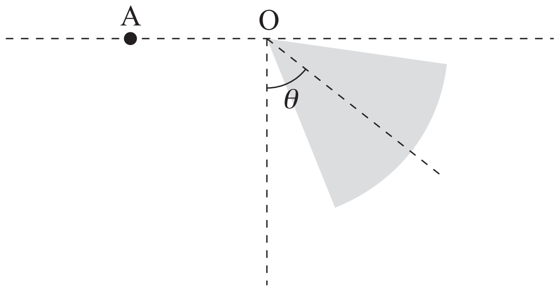

Fig. 4.2

(iii) Show that $\dot{\theta}^2 = \frac{4g}{\pi a}(2\cos\theta + 1)$. **[4]**

(iv) Find the greatest speed attained by any point on the lamina. **[4]**

(v) Find an expression for $\ddot{\theta}$ in terms of θ, a and g. **[2]**

The lamina strikes a fixed peg at A where $AO = \frac{3}{4}a$ and is horizontal, as shown in Fig. 4.2. The collision reverses the direction of motion of the lamina and halves its angular speed.

(vi) Find the magnitude of the impulse that the peg gives to the lamina. **[4]**

(vii) Determine the maximum value of θ in the subsequent motion. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

Monday 10 June 2013 – Morning

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4764/01 Mechanics 4

PRINTED ANSWER BOOK

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OCR supplied materials:
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- MEI Examination Formulae and Tables (MF2)

Other materials required:
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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

Section A (24 marks)

1 (i)	
	(answer space continued on next page)

1 (i)	**(continued)**

1 (ii)	
2 (i)	

2 (ii)	
	(answer space continued on next page)

2 (ii)	**(continued)**
2 (iii)	

Section B (48 marks)

3 (i)	
	(answer space continued on next page)

3 (i)	**(continued)**

3 (ii)	
3 (iii)	

3 (iv)

(answer space continued on next page)

3 (iv)	**(continued)**

4 (i)	

4 (ii)	
4 (iii)	

4 (iv)

4 (v)

4 (vi)	

4 (vii)	

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4764:** Mechanics 4

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

Question			Answer	Marks	Guidance	
1	(i)		$\frac{dm}{dt} = k \Rightarrow m = kt + c$	B1		
			conditions $\Rightarrow m = m_0 + kt$	B1		
			$\left(\frac{d}{dt}(mv) = 0 \Rightarrow\right) \quad mv = m_0 v_0$	M1 A1	Momentum equation	Or derive a differential equation in only two variables
			$v = \frac{m_0 v_0}{m} = \frac{m_0 v_0}{m_0 + kt}$	A1		
			$x = \int \frac{m_0 v_0}{m_0 + kt} dt$	M1	Integrate their expression for v	
			$= \frac{m_0 v_0}{k} \ln(m_0 + kt) \quad (+ c_2)$	A1		
			conditions $\Rightarrow 0 = \frac{m_0 v_0}{k} \ln m_0 + c_2$	M1	Use initial conditions	
			so $x = \frac{m_0 v_0}{k}(\ln(m_0 + kt) - \ln m_0)$			
			$= \frac{m_0 v_0}{k} \ln\left(\frac{m_0 + kt}{m_0}\right) = \frac{m_0 v_0}{k} \ln\left(1 + \frac{kt}{m_0}\right)$	E1		
				[9]		
1	(ii)		$kt = 2m_0 \Rightarrow t = \frac{2m_0}{k} \Rightarrow v = \frac{1}{3} v_0$	B1	Follow through their $v = f(t)$	**SC** If $kt = m_0$ Award B1 either correct on follow through
			$x = \frac{m_0 v_0}{k} \ln 3$	B1	Ft	
				[2]		
2	(i)		$BC = 2 \times 0.5 \sin \frac{1}{2}\theta$	E1		
				[1]		

Question			Answer	Marks	Guidance	
2	(ii)		$V = -0.5g \cdot 0.25\sin\theta +$ $\frac{1}{2}\cdot 2(\text{BC}-0.5)^2 + \frac{1}{2}\cdot 2(\text{BD}-0.5)^2$ $\text{BD}^2 = 1^2 + 0.5^2 - 2\times 1\times 0.5\cos\theta = 1.25 - \cos\theta$ $V = -1.225\sin\theta + (\sin\frac{1}{2}\theta - 0.5)^2 +$ $(\sqrt{1.25-\cos\theta} - 0.5)^2$ $\frac{\mathrm{d}V}{\mathrm{d}\theta} = -1.225\cos\theta + 2(\sin\frac{1}{2}\theta - 0.5)(\frac{1}{2}\cos\frac{1}{2}\theta) +$ $2\left(\sqrt{1.25-\cos\theta} - 0.5\right)\left(\frac{\sin\theta}{2\sqrt{1.25-\cos\theta}}\right)$ $= -1.225\cos\theta + \sin\frac{1}{2}\theta\cos\frac{1}{2}\theta - 0.5\cos\frac{1}{2}\theta +$ $\sin\theta - \frac{0.5\sin\theta}{\sqrt{1.25-\cos\theta}}$ $= 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25-\cos\theta}} - 0.5\cos\frac{1}{2}\theta$	M1 M1 B1 A1 M1 M1 A1 E1 **[8]**	GPE At least one EPE term oe Differentiate Use of chain rule one EPE term correct Complete argument	
2	(iii)		$\theta \approx 1.2$ and 4.1 Stable and unstable respectively at $\theta \approx 1.2$, $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ increasing because the graph shows that $\mathrm{f}'(\theta)$ is positive so V minimum hence stable at $\theta \approx 4.1$ $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ decreasing because the graph shows that $\mathrm{f}'(\theta)$ is negative, so max. so unstable	B1 B1 M1 A1 **[4]**	Both Consider gradient, relating f to $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ Clear evidence from the graph	Allow B1M1A1 from 1.1 and/or 4.05

Question			Answer	Marks	Guidance	
3	(i)		$2\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{2v^3+4v}{v}-6v$	M1 A1	N2L	
			$\frac{\mathrm{d}v}{\mathrm{d}t}=v^2-3v+2=(1-v)(2-v)$	E1		
			$\int\frac{1}{(1-v)(2-v)}\mathrm{d}v=\int\mathrm{d}t$	M1	Separate	
			$\int\left(\frac{1}{1-v}-\frac{1}{2-v}\right)\mathrm{d}v=\int\mathrm{d}t$	M1	Partial fractions	
			$-\ln\lvert 1-v\rvert+\ln\lvert 2-v\rvert=t+c$	A1 A1	LHS RHS	
			$t=0, v=0\Rightarrow\ln 2=c$	M1	Use condition	
			$t=\ln(2-v)-\ln(1-v)-\ln 2$	M1	Rearrange	
			$=\ln\frac{2-v}{2(1-v)}$	E1		
				[10]		
3	(ii)		$\Rightarrow\frac{2-v}{2(1-v)}=\mathrm{e}^t\Rightarrow 2-v=2\mathrm{e}^t-2\mathrm{e}^t v$	M1	Rearrange	
			$v=\frac{2(\mathrm{e}^t-1)}{2\mathrm{e}^t-1}$	A1		
				[2]		
3	(iii)		$v=0.8\Rightarrow P=2\times0.8^3+4\times0.8=4.224$	E1		
			$t=\ln\frac{2-0.8}{2(1-0.8)}=\ln 3\approx1.10$	B1		
				[2]		

Question			Answer	Marks	Guidance	
3	(iv)		$2\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{4.224}{v}-6v$	B1	N2L	
			$\int \frac{2}{\frac{4.224}{v}-6v}\mathrm{d}v=\int \mathrm{d}t$	M1	Separate	
			$\int \frac{v}{2.112-3v^2}\,\mathrm{d}v=\int \mathrm{d}t$	M1		
			$-\frac{1}{6}\ln\left\|2.112-3v^2\right\|=t+c_2$	A1	LHS	
				A1	RHS	
			$2.112-3v^2=A\mathrm{e}^{-6t}$			**Alternate**
			$t=\ln 3, v=0.8\Rightarrow 2.112-3\times 0.8^2=A\mathrm{e}^{-6\ln 3}$	M1	Use condition to find constant	
			$A=139.968$	A1		
				M1	Rearrange to make v the subject	$t\to\infty\Rightarrow 2.112-3v^2\to 0$
			$v=\sqrt{0.704-46.656\mathrm{e}^{-6t}}$	A1	Correct	
			$t\to\infty\Rightarrow v\to\sqrt{0.704}\approx 0.839$	A1	Ft their expression for v	
				[10]		
4	(i)		$\rho=\frac{m}{\frac{1}{2}a^2\cdot\frac{\pi}{3}}$	B1	Or let ρ = 1 without lose of generality	
			element with radius x and 'width' δx :			
			$\delta m=\rho x\frac{\pi}{3}\delta x\Rightarrow \delta I=\rho x\frac{\pi}{3}\delta x\cdot x^2$	M1		
			$=\frac{2m}{a^2}x^3\delta x$	A1	Ft their ρ	
			$I=\int_0^a \frac{2m}{a^2}x^3\,\mathrm{d}x$	M1	Integrate	
			$=\frac{2m}{a^2}\left[\frac{x^4}{4}\right]_0^a$	A1		
			$=\frac{2m}{a^2}\left(\frac{a^4}{4}\right)=\frac{1}{2}ma^2$	E1		
				[6]		

Question			Answer	Marks	Guidance	
4	(ii)		$\frac{2a}{\pi}$	B1 **[1]**		
4	(iii)		$\frac{1}{2}I\dot{\theta}^2 - mg\left(\frac{2a}{\pi}\right)\cos\theta = -mg\left(\frac{2a}{\pi}\right)\cos\frac{2}{3}\pi$ OE $\frac{1}{2}ma^2\dot{\theta}^2 = 2mg\left(\frac{2a}{\pi}\right)\left(\cos\theta + \frac{1}{2}\right)$ $\dot{\theta}^2 = \frac{4g}{\pi a}(2\cos\theta + 1)$	M1 A1 A1 E1 **[4]**	Energy Two correct terms All correct	RHS: or $mg\left(\frac{2a}{\pi}\right)\cos\frac{1}{3}\pi$
4	(iv)		Max $\dot{\theta}$ when $\cos\theta = 1$ $\Rightarrow \dot{\theta}^2 = \frac{12g}{\pi a}$ Speed max. furthest from axis, so max speed $= a\sqrt{\frac{12g}{\pi a}} = \sqrt{\frac{12ag}{\pi}}$	M1 A1 M1 A1 **[4]**	oe	
4	(v)		$2\ddot{\theta}\dot{\theta} = \frac{4g}{\pi a}(-2\sin\theta\dot{\theta})$ $\ddot{\theta} = -\frac{4g}{\pi a}\sin\theta$	M1 A1 **[2]**	Differentiate with respect to time Or use $C = I\ddot{\theta}$	May be seen in (iv) May be seen in (iv)
4	(vi)		$J \cdot x = \pm I \cdot \frac{1}{2}\omega \pm I \cdot \omega$ $J \cdot \frac{3}{4}a = \frac{1}{2}ma^2\left(\frac{1}{2}\omega\right) - \frac{1}{2}ma^2(-\omega)$ $\theta = (-)\frac{1}{3}\pi \Rightarrow \omega^2 = \frac{4g}{\pi a}\left(2\left(\frac{1}{2}\right)+1\right) \Rightarrow \omega = \sqrt{\frac{8g}{\pi a}}$ $J = m\sqrt{\frac{8ag}{\pi}}$	M1 A1 B1 A1 **[4]**		

Question			Answer	Marks	Guidance	
4	(vii)		$\frac{1}{2}I\left(\frac{1}{2}\sqrt{\frac{8g}{\pi a}}\right)^2 - mg\left(\frac{2a}{\pi}\right)\cos\frac{1}{3}\pi = -mg\left(\frac{2a}{\pi}\right)\cos\theta$	M1 A1		
			$\Rightarrow \theta = \cos^{-1}\frac{1}{4} \approx 1.32$	A1 **[3]**	CAO	

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GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4764 Mechanics 4

General Comments

The standard of performance was very high. As usual, candidates showed their skills in solving differential equations and manipulating complicated expressions, and most of them demonstrated a solid knowledge of the techniques and concepts required.

Comments on Individual Questions

1) ***Variable mass***

(i) Most candidates found the given answer correctly, though many did far more work than required. Examiners do not require the *δt* derivation of the equation of motion in variable mass questions of this type where in the direction of motion of the truck there is no external force and *mv* is the total linear momentum of the system. In such a case we would accept $\frac{d(mv)}{dt}=0$ without justification. The best candidates having started by deriving or stating $\frac{d(mv)}{dt}=0$ or equivalent then went on to say $mv = m_0v_0$ or equivalent. Some candidates passed through a stage with $m\frac{dv}{dt}+v\frac{dm}{dt}=0$, or equivalent, without recognising that this gives $\frac{d(mv)}{dt}=0$ and went on unnecessarily to solve a differential equation involving *v* and *t*.

Many candidates did not give enough justification for the expression for mass in terms of *t*, or enough steps in their approach to the final given answer.

(ii) A large majority of candidates misinterpreted this request as asking for the two values at the moment when $m = 2m_0$ rather than $3m_0$. A special case was added to the mark scheme to allow 1 out of 2, but candidates are reminded of the need to read stems carefully to avoid this sort of error.

2) ***Equilibrium***

(i) This was done well by most candidates. It is difficult to give a good justification for this given answer purely symbolically. The best solutions included a diagram to show where the various factors of ½ came from.

(ii) The general approach was well understood by most candidates. The derivation of an expression for BD was done very well, with most using the hint from part (i) effectively. Many candidates chose to expand their expressions for the GPE terms before differentiating rather than use the chain rule; this led, in some cases, to copying and sign errors that might otherwise have been avoided.

(iii) Most candidates found the values of *θ* at the potential points of stability from the graph and correctly determined whether or not they were stable. However, many could not provide justification, or justified their choice by evaluating the second differential of *V* at each value rather than using the graph as directed. For a solution to be awarded full marks it had to include a clear use of information from the graph, an explicit relationship between f(*θ*) and *V* and a brief mention of the conditions for stability.

3) ***Variable force***

Parts (i) (ii) and (iii) were done very well by the majority of candidates, with careful and precise work.

(iv) Most candidates integrated a correct Newton's second law equation to obtain an equation in *v* and *t*. However, many then used incorrect conditions, usually *t* = 0 rather than ln 3 when *v* = 0.8. Some did define a new variable for the time at constant power, for example *T* = *t* – ln 3, and this was awarded full marks when done carefully.
Many candidates did not derive an equation for *v* in terms of *t* as requested, instead finding the limiting value of *v* by consideration of their implicit equation.
This last step of finding the limiting value was performed very well by the majority.

4) ***Rotation***

(i) This proof was done very well by most candidates. Some chose to take the mass per unit area to be 1, but only those that did so explicitly were awarded full marks.
A few candidates used the standard bookwork to derive the moment of inertia of a disc and then argued from symmetry to get the given answer. This method was only awarded full marks when carefully justified.

(ii) Most candidates used the formula in MF1 correctly. Values of $\frac{a}{\pi}$ and $\frac{2a}{3}$ were followed through for all but given answers.

(iii) This was well answered by the majority of candidates. To obtain full marks needed careful manipulation to the given answer without leaving out too many steps towards the end.

(iv) This was generally well done with only a few candidates stopping once they had a value for $\dot{\theta}$.

(v) Two approaches were seen each in approximately half the scripts.

1. Differentiation of the expression in (iii). This was often done well, but many candidates left out the $\dot{\theta}$ term on the RHS.
2. From $C = I\ddot{\theta}$. Many candidates had the sign of the couple as positive rather than negative, but otherwise this was done well.

(vi) Most candidates who attempted this could set up an equation of the form $Jx = I\Delta\omega$, but many errors were made, usually in evaluating *ω* or taking both values of *ω* as having the same sign.

(vii) Very few correct solutions were seen to this question. Many candidates who attempted it did not include both GPE terms and therefore scored zero. Those who included all three terms often had an error, for example the wrong value of *ω*, sign errors in the GPE terms or manipulation errors in the solution of their equation,

Unit level raw mark and UMS grade boundaries June 2013 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0